Remote synchronization, 2 ways:

<u>I. Photon → atom (or other "storage" qubit)</u>

1. Start with (degenerate) photons in the state:

$$|\psi\rangle_{photon} = \frac{1}{\sqrt{2}}\left[|H\rangle|V\rangle - |V\rangle|H\rangle\right]$$

By symmetry, this state does not time-evolve (even if the photons weren't degenerate).

2. One photon is sent to Alice, the other to Bob (or Alice may start with both, and send one to Bob).

3. Alice and Bob both transfer their photon states to their (**non-degenerate**) storage qubits. The state of the two storage qubits, also in a singlet state, does not time-evolve: $|\psi\rangle_{atom} = \frac{1}{\sqrt{2}}\left[|g\rangle_A|e\rangle_B - |e\rangle_A|g\rangle_B\right]e^{i(\omega_g+\omega_e)t}$

4. At time t = 0, Alice measures her qubit to be in the state: $\frac{\langle g|_A + \langle e|_A}{\sqrt{2}}$

This projects Bob's qubit into the time-evolving state:

$$|\psi(t)\rangle_{Bob} = \frac{1}{2}\left[|e\rangle_B e^{i\omega_e t} - |g\rangle e^{i\omega_g t}\right]$$

5. When Bob measures his qubit in the state at time t = T, he will find the state rotated:

$$= \left|\frac{\left(\langle g|_B - \langle e|_B\right)}{\sqrt{2}}|\psi(T)\rangle_{Bob}\right|^2 = \left|\frac{-1}{2\sqrt{2}}\left(e^{i\omega_g T} + e^{i\omega_e T}\right)\right|^2 = \frac{1}{4}\left[1 + \cos\Delta\omega T\right]$$

$\rightarrow$ Knowledge $P, \Delta\omega \Rightarrow T$

The problem:
Unless Alice and Bob transfer their photon state to the non-degenerate storage qubits at *precisely the same time*, there will be some time during which they have a photon+qubit system, which ***is*** time-evolving. This time-evolution could be corrected for, but only if Alice and Bob already knew the exact time shift between their local clocks, which is the problem they were initially trying to solve.

II. Photons all the way

1. Start with *non-degenerate* photons in the singlet state. By symmetry, this state still does not time-evolve:

$$\frac{1}{\sqrt{2}}\left[|H\rangle|V\rangle - |V\rangle|H\rangle\right] e^{i(\omega+\omega)t}$$

2. One photon is sent to Alice, the other to Bob (or Alice may start with both, and send one to Bob).

3. At time t = 0, Alice measures her photon in the state: $\frac{{}_A\langle H| - {}_A\langle V|}{\sqrt{2}}$

Note the projection is done by the act of passing through a polarizer, which may be much quicker than a detector. This projects Bob's photon into the time-evolving polarization state:

$$\frac{1}{2}\left[|H\rangle_B e^{i\omega t} + |V\rangle_B e^{i\omega t}\right] = \frac{e^{i\omega t}}{2}\left[|H\rangle + |V\rangle e^{i(\omega-\omega)t}\right]$$

4. Bob measures his photon at time t = T, his probability of detection will depend on T and $\Delta\omega$. Again, it is the passage through the polarizer that counts, not the detection speed:

$$P = \left| \frac{\langle H| + \langle V|}{\sqrt{2}} \left(\frac{e^{i\omega t}}{2}\right)\left(|H\rangle + |V\rangle e^{i\Delta\omega T}\right)\right|^2$$

$$= \frac{1}{4}\left(1 + \cos\Delta\omega T\right)$$

Knowledge of $P, \Delta\omega \Rightarrow T$

Problem 1:

The elapsed time from when Alice's photon goes through her polarizer, and Bob's through his, depends on the precise optical path delay (from Alice to the source and back out to Bob), which presumably we don't know.

However, we might then think to use this method to accurately measure this optical path *length*, without needing overly fast detectors:

$$P \Rightarrow \frac{1}{4}\left[1 + \cos \Delta\omega \left(\frac{n_B X_B - n_A X_A}{c}\right)\right]$$

Problem 2:

The evolution phase factor is actually in time and space: $e^{i(\omega t - kx)}$

If we sit at a given place in space, we still get a time-dependent oscillation, i.e., a beating. One solution is to use the time-correlation of the photons' themselves to restrict this. However, the photons are only coincident to within a time $\Delta t \sim 1/\delta\Omega$:

If the spectral width $\delta\Omega$ is large, then $\Delta\omega$ becomes uncertain.

If the spectral width is small, then Δt becomes too large, and the temporal beating washes out the signal. $\int_0^{\Delta t} dt \cos \Delta\omega t \Rightarrow 0$

The other solution is to use precise knowledge of the detector firing (and the relative clock timing) to essentially perform a phase-locked measurement. But again, this requires one to have already solved the clock-synchronization problem.